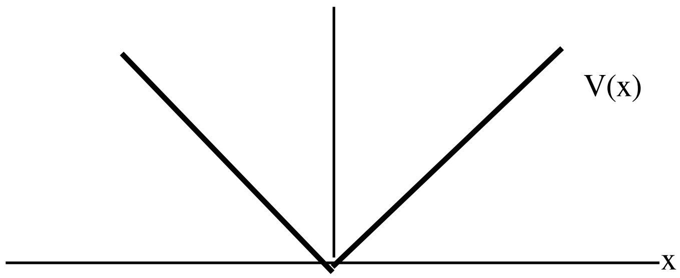
Final Physics 9HC spring 2000

Use a law blue book. Closed book. Two 3"x5" note cards. PRINT your name on your blue book. To get credit, you must show your work. Be neat, clear, and organized. If we can't read it or figure it out, we can't give you any points. Since a lot of these have "Explain" parts, take time to write clearly.

1. (25) Consider the linear potential V(x)=A|x| with A a known constant.



a) i) In its lowest energy state, a classical particle would just sit at x=0. Suppose that a quanton were put in a state that had it *highly* localized in an arbitrarily small region near x=0. Would that be the lowest energy state for the quanton? Explain.
ii) If you were going to do the problem correctly and find the value of the lowest energy E_1 and the precise shape of the wave function, you would have to solve the time independent Schroedinger equation for this problem. Precisely what is the form that equation takes for this problem?
Before doing a hard problem like that, it is a good idea to try to understand it in simpler terms. Suppose that the lowest energy state labeled n=1 has an energy of E_1.
iii) In terms of E_1 and A, what are the x values of the n=1 turning points?
iv) On a graph with the turning points indicated, sketch the lowest energy state wave function.
v) Sketch the n=6 wave function. (Keep in mind that it will have different turning points.)
vi) For the harmonic oscillator, the energy levels increase proportional to n. For this linear potential, will they increase more or less rapidly than that with n?
vii) In the forbidden region of the step potential, the wave function decays exponentially with one power of x in the exponent $\propto$ exp(-Cx). For the harmonic oscillator, the decay in the forbidden region is faster with an x^2 in the exponent $\propto$ exp($-Cx^2$). For this linear potential, will the decay rate be faster than the harmonic oscillator, between the harmonic oscillator and the step potential, or slower then the step potential? Explain.

b) For t<0, the potential is as given above. Let x_T be the right-most n=1 turning point. At t=0, the potential is modified so that V(x)=0 for x>$2x_T$. For x<$2x_T$, V is unchanged.
i) Sketch the potential for t>0 with E_1 and the turning points indicated.
For t $\le$ 0, the quanton is in the lowest energy state, but after the potential changes, the wave function starts changing as t increases.
ii) Explain roughly what happens to the wave function as t increases from t=0.
iii) After a very long time, what will the wave function look like for x in the region between the original turning points? Specifically, will it be roughly the same size as it was at t=0 but considerably modified in shape due to the changed potential, or modified in shape and *very* much smaller in size, or modified in shape so that it is more concentrated near x=0?

2. (10) Now consider the harmonic oscillator problem with its equal spacing of energy levels. Assume that in each case, things are arranged so that the lowest energy state with one particle in the potential is E_1. I will now put various pairs of particles in the potential. In all cases, the two particles interact with the harmonic oscillator potential, but there is no additional potential associated with a direct interaction between the two particles. In each case, give total energy of the lowest energy state of the system the with two particles. Provide one or two sentences to justify your choice.

a) Two electrons with their spins in opposite directions.
b) Two electrons with their spins in the same direction.
c) A proton and a neutron (each of which is spin-1/2 particle) with their spins in the same direction.
d) Two identical spin one particles with their spins in the same direction.
e) Two spin one particles identical except that one has $S_z=\hbar$ and the other has $S_z=0$.

3. (25) In magnetic resonant imaging (MRI) a person is placed in a strong magnetic field, and the behavior of the spins of atomic nuclei are used to make images of the person's inner workings. Suppose that in the absence of fields other than the applied one, the energy levels of the spin-1/2 hydrogen nucleus (just a proton) are -1eV and +1eV. (In reality, the energies are much smaller than that.) Now in addition to the applied field from the machine that gives those levels, there is another magnetic field in a different direction acting on the proton that comes from a nearby atom. In that case, in addition to the diagonal elements of the Hamiltonian $H_{11} = -H_{22} = 1\text{eV}$ that come from the applied field, there are also off-diagonal elements $H_{12} = -H_{21} = iA$ with A real. Using the MRI machine, I observe that I must supply a photon of energy 3eV to flip the spin from its low energy state to its higher energy state. What is the value of A? From such shifts, one can learn about the environment of the nuclei and make the detailed MRI pictures. Hint: You will want to start by calculating the energy levels in terms of A. From those and the given data, A can be easily determined.

4. (20) Compare stationary states of the infinite square well and the *bound states* of the hydrogen atom. (That is, do not use the unbound electron–proton states in answering the questions.)

i) In the spectrum of possible photon emissions, is there a longest possible wavelength for the square well? for hydrogen? Explain.
ii) For each case, is there a shortest possible wavelength? Explain.

5. (20) Consider the infinite square well potential confining a particle between x=0 and x=L. You are told that the particle may be in either the n=1 (lowest energy) level or in the first excited n=2 level. You have a detector that has a width $\Delta x = L/10$. You get to make one measurement with your detector. If you position the center of the detector at x and the particle is in the range $[x-\Delta x/2, x+\Delta x/2]$, you will detect it. Then depending on whether you detect the particle or not, you bet on whether the particle was in the n=1 or the n=2 level before you tried to detected it. To maximize your chance of winning the bet, where should you place the detector? Although you could do this problem with a tedious analytical calculation, you do not get that much time. Just tell me your choice for the position x of the center of the detector and use a picture to explain why it is the best choice.