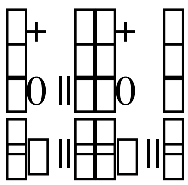
Lecture 11 error discussion

Most of you were in class on Tuesday and saw a spectacular conceptual error on my part. Let's look at the error, how to correct it, and a better example of the point that I had hoped to make. Of course the point that has actually been made and that you will now remember concerns the fundamental issue of when to add amplitudes and when to add probabilities. Fortunately this is a very important issue, so that the cost in embarrassment to me has been toward the worthy goal of fundamental understanding.

The problem

Consider the set up I used in class



S *T*

and the question "What is the probability that a spin-1 quanton prepared by passing through S with just the top open will pass through T with the top two open?"

Discussion

Is this a single process? If yes, then there are two ways for it to happen, and the amplitudes should be added. If more than one process is involved, then the probability for each needs to be computed before the probabilities are added. So which is it? A spin one quanton that makes it out of T may have its spin in the +1 state or in the 0 state. These are physically distinct states. We can do a measurement *e.g.* with another T apparatus to determine the final state spin. Thus "makes it out of T" is not a specification of a final state. "Makes it out of T in the +1 state" is a specification of a final state as is "makes it out of T in the 0 state." This we need to compute the amplitudes and then the probabilities of each of these two processes before adding the probabilities. It is not like the two-slit problem because in that case, there was no extra physical property analogous to the spin state that tells you which slit the quanton went through.

Error

I incorrectly treated them as one process and wrote

$P = \left|\langle +T|+S\rangle + \langle 0T|+S\rangle\right|^2.$

The notation alone indicates an error: the two amplitudes do not have the same final state.

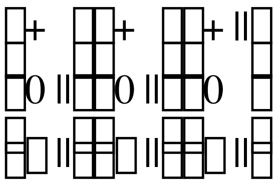
Correction

The correct procedure accounting for two physically distinct process is

$P = P_+ + P_0 = \left|\langle +T|+S\rangle\right|^2 + \left|\langle 0T|+S\rangle\right|^2$

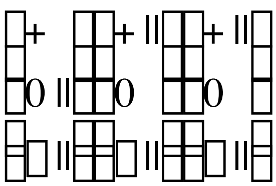
So this illustrates a point about when to add amplitudes before computing probabilities and when to compute probabilities and then add.

New problem



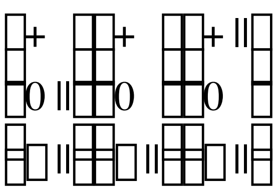
S *T* *S*

$$P_+ = \left|\langle 0S|+T\rangle\langle +T|+S\rangle\right|^2 = \left|\frac{1}{2}\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)\right|^2 = \left|\frac{1}{4\sqrt{2}}\left(\sqrt{2}+1\right)\right|^2$$



S *T* *S*

$$P_0 = \left|\langle 0S|0T\rangle\langle 0T|+S\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right)\right|^2 = \left|-\frac{1}{4\sqrt{2}}2\right|^2$$



S *T* *S*

$$P_2 = \left|\langle 0S|0T\rangle\langle 0T|+S\rangle + \langle 0S|+T\rangle\langle +T|+S\rangle\right|^2$$

$$= \left|\frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right) + \frac{1}{2}\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)\right|^2 = \left|\frac{1}{4\sqrt{2}}\left(\sqrt{2}-1\right)\right|^2$$

Note that

$P_2 < P_0 < P_+$

Thus relative to either the first or second process each with one open hole, opening the second hole in the third process increases the available paths but *decreases* the probability that the quanton gets out of the second S apparatus!