

Midterm Physics 9HC spring 2000

Use a law blue book. Closed book. One 3"x5" note card. PRINT your name on your blue book. To get credit, you must show your work. Be neat, clear, and organized. If we can't read it or figure it out, we can't give you any points.

$hc = 1240 \text{ eV nm}$, $c = 3 \times 10^8 \text{ m/s}$, $\hbar = h/(2\pi)$ $h = 6.63 \times 10^{-34} \text{ Js}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$\sin A + \sin B = 2 \sin[(A+B)/2] \cos[(A-B)/2]$$

1. (25) Consider an infinitely long string clamped at $x=0$ and $x=L=2\text{m}$. The wave speed on the string is 10m/s . Initially consider the portion of the string between the clamps.

a) What are the frequencies of the first two standing waves?

b) For each of them, write an expression for the string displacement $y(x,t)$ when the maximum displacement is 2cm .

c) Show by explicit calculation that the lowest frequency mode is a solution to the wave equation in the x interval $(0,L)$.

d) Now consider the whole string. For $t < 0$, the next-to-lowest-frequency mode with maximum displacement 2cm is oscillating in the $[0,L]$ interval and the rest of the string has $y=0$. At the instant $t=0$ when the string in $(0,L)$ displaced according to the oscillation described and is motionless, the clamps are suddenly removed. Draw the configuration of the string 1s later. Include numerical labels on the x -axis. *Explain* carefully why your picture is correct and why it represents a solution to the wave equation. (It is not easy to derive the answer deductively. It is OK to assert an answer and then explain why it is correct.)

2. (25) Consider the following list:

i) single slit diffraction of sound waves

ii) 2-slit interference of small amplitude water waves

iii) beats from a pair of tuning forks

iv) the expression for the general solution to the 1-dimensional wave equation

v) the Doppler shift

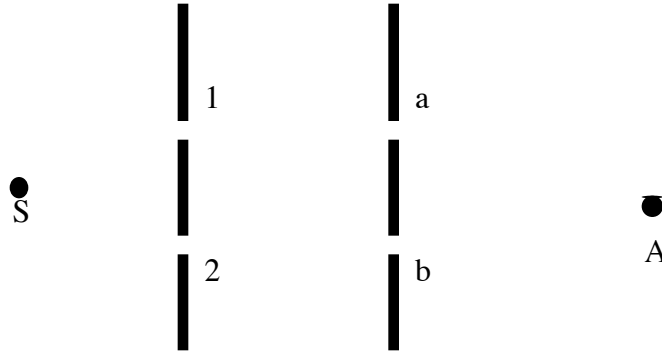
vi) 2-slit interference for electrons

a) Which of the above does *not* involve the superposition principle? (Explain)

b) Consider (ii) and (vi). Explain qualitatively the basic physics of how the superposition principle leads, in both cases, to the interference pattern.

c) Yet there is an important difference. Describe the quantities to which the superposition is applied in the two cases (ii) and (vi). In what way are these quantities fundamentally different?

3. (25) Consider the probability $P(A,S)$ that a quanton emitted at S is observed at A.



a) Write the probability amplitude $\langle A|S \rangle$ for the process in terms of the amplitudes $\langle 1|S \rangle$, $\langle a|1 \rangle$, $\langle A|a \rangle$, etc.

Suppose that $\langle 1|S \rangle = \langle 2|S \rangle = \langle A|a \rangle = \langle A|b \rangle = 1/\sqrt{2}$

$$\langle a|1 \rangle = ig \quad \langle b|1 \rangle = -ig \quad \langle a|2 \rangle = -g \quad \langle b|2 \rangle = g \quad \text{with } g = 0.1$$

b) What is the value of $P(A,S)$?

c) Of all the ways to have the holes open or closed, what is the largest $P(A,S)$ that can be achieved? What arrangement of open and closed holes gives that? Is there more than one such arrangement?

4. (25) A beam of coherent electromagnetic radiation is directed at two narrow slits separated by 0.1 mm. On a screen 10m from the slits, there is an interference pattern with the maxima separated by 6.2 cm. The peak intensity is 10W/m^2 .

a) Your friend, who has studied classical waves but not quantum mechanics, says "See, this shows that light is a wave with wavelength λ ." What should λ equal to make that sentence correct so far as it goes?

b) You answer "Well, yes, but it is better described by a quantum photon picture for light." After some discussion, your friend agrees to be persuaded of the particle aspect of light if, without changing anything, you can demonstrate a photon counting rate on the screen of less than 10/s. The smallest detector you can build has an area of 1 (nm)^2 . With that detector, what is the photon counting rate in the forward, $y=0$ in our usual notation, direction?

c) What is the minimum distance y you need to move the detector off center to satisfy your friend?