Semifinal 7B, SS2 2007

September 14, 2007

Question 1a) [30%]

q	10	Correctly found the jar was half full on the 6th day.
t	8.5	Mistakenly found $k = 2$ rather than $k = \ln 2$ and used $N(t) = N_0 e^{kt}$ and thus obtained 6.65 days.
s	7.0	Did not find $t = 6$ days, but did write down $N = N_0 e^{\ln 2\, t}$ or equivalent.
u	6.8	Wrote three series of numbers down, one of which was "1,2,4,8,16,32,64" and wrote "between six and seven days" with no further explanation.
x	6.2	Gave the answer: "t = (0.5) * (7 days) = 3.5 days". i.e. thought growth was linear.
y	3.0	Little or no working.
z	0.0	Blank or effectively blank.

Question 1b) [30%]

q	10	All correct. The answer could either be expressed in base 2 or base e. The answer was $F(t) = \frac{1}{128}2^t = \frac{1}{128}e^{\ln 2\, t}$
r	9.2	All correct except for a minor calculation error.
s	7.4	Determined the time constant correctly, but did not find F_0 or found it incorrectly.
u	6.7	Did not get either the time constant or the initial number F_0 correct.
w	6.3	Answered $F(t) = (1/2) \times (7 - t)$ rather than the correct $F(t) = \left(\frac{1}{2}\right)^{7-t} = 2^{t-7}$. There is a huge difference between multiplication and exponentiation!
x	4.0	An exponential *decay* function was given for $F(t)$ and either F_0 or k were not determined correctly.
y	2.0	Serious errors or inconsistencies.
z	0.0	Blank or essentially blank.

Question 1c) [30%]

q	10	All correct, with a correct explanation.
r	9.2	Correct answer given, including a correct explanation but the response included an erroneous or misleading statement.
t	8.4	Correctly identifies the doubling time as one day but fails to explain why it takes longer to grow by a factor of *e*.
u	6.8	Correct answer with very bad explanation.
x	3.0	Correct answer not given (it really came down to the fact $2.818\ldots > 2$, a fact every college student should know).
y	3.0	Correct answer with no explanation. It says in the question to explain!
z	0.0	Blank or essentially blank.

Question 1d) [10%]

q	10	All correct.
y	6.5	Answer showed student knew what they were doing, but they wrote completely non-sensical statements e.g. $2\pi = 1$ year. The *meaning* is clear, but this laziness is unacceptable.
x	2.0	Student demonstrated they did not know how long it takes the Earth to orbit the Sun.
z	0.0	Blank or essentially blank.

Common errors for the various problems are discussed below. However, it is worth taking a moment to discuss some general errors in method and analysis that are common to all problems:

- While arithmetic errors are generally considered to be minor, if they yield a result that is grossly numerically incorrect (e.g. the final energy of the system is much greater than the initial), they may be considered to be more serious.
- Ridiculous algebraic errors may be considered to be serious. For example, $1/a + 1/b \neq \frac{1}{a+b}$.
- Most calculations should have units. Calculations that clearly yield the incorrect units (aka, I = 6v) might be considered very serious errors.
- Making up equations such that they incorporate all the constants and variables in a given problem is discouraged. This approach is often effective in Numerology, but less so in physics.
- "Cameling," or constructing a solution from bits and pieces of DLs and FNTs that dont really fit together is discouraged.

- Attributing a systems physical behavior to nonsensical or irrelevant physical laws might be a serious error.
- Grossly misinterpreting the physical meaning of calculations, or the consequences of physical laws may be considered a serious error.
- Illegible solutions cannot be verified.

For both problems 2 & 3, we are looking for solutions that show a fundamental grasp of Ohms Law, $\Delta V = -IR$. In summary, this means that given a perfect voltage source, a battery, reducing resistance increases current flow; increasing resistance impedes the flow of current. Adding resistors in series increases resistance; resistors in parallel add in reciprocal, so their combined resistance is smaller than any of the elements.

Question 2a) [5%]

The ΔV of each of the lightbulb's is equal to the full ΔV of the battery (except of course R, which is not connected while the switch is open). The power used by each resistor is $P = -(\Delta V)^2/r$, so the larger r is the smaller P is (again, with the exception of the resistor that is not connected).

$$|P_R| < |P_{3R}| < |P_{2R}|$$

Q	10	Perfect.
R	9.5	A minor error accompanied by a convincing explanation.
S	8	A substantial error accompanied by an explanation
T	7	Invoking Ohms law, but to the wrong conclusion.
U	6	A more substantial error, for example showing R1 to light up, despite the fact that when the switch is open, it is not connected to a voltage source.
W	2	All but blank; nonsensical.
X	0	Blank or totally non-pertinent.

Question 2b) [15%]

R gets brighter because it is connected to the battery. The voltage difference across the other two resistors do not change, because ΔV stays the same for them.

q	10	Perfect
r	8.9	The solution is correctly chosen and the explanation is correct but has minor flaws.
s	8.3	Solution is chosen correctly but argument is weak.
t	7	Demonstrates some knowledge of Ohm's law, but misinterprets its implications for resistors in parallel. For example, the solution might note that P_R increases since it is now connected to the battery, but suggests that P_{2R} and P_{3R} must decrease to compensate.
u	6	More serious errors. The explanation demonstrates little understanding of Ohm's law, or the circled answers are correct but there is no explanation.
w	2	All but blank, non-sensical
z	0	Blank

Question 2c) [15%]

$P_{\text{battery}} = |P_{\text{resistors}}|$. The simplest answer is that energy is conserved. Long calculations that ultimately show that energy is conserved (aka, $P_b = (\Delta V)I; P_r = P_{R1} + P_{2R} + P_{3R}$) are less desirable but correct, so long as they are carried out to completion. A correct answer must invoke or explicitly show energy conservation.

q	10	Perfect. The answer is correct and clearly and elegantly invokes energy conservation.
r	9	The correct answer involves energy conservation, but with some confusion or incompleteness.
s	8.7	The answer indicates elements of energy conservation, but does not quite spell it out completely. For example, the solution correctly states that ΔV around the entire circuit is zero, but fails to complete the energy argument by discussing current.
t	6	Solution indicates an understanding of Ohm's law, but perhaps misinterprets the question. For example, one might say $P_{\text{battery}} < \|P_{\text{resistors}}\|$ since R was removed from the circuit, implying that if the switch was closed $P_{\text{battery}} = \|P_{\text{resistors}}\|$
u	3.5	The solution indicates little understanding of Ohm's law or energy conservation.
w	2	All but blank
x	0	Blank

Question 2d) [10%]

In short, ΔV_2, ΔV_3 do not change, and of course the resistances are fixed, so P_{2R} and P_{3R} do not change when we close the switch. P_R, now in circuit with the battery, lights up, and so the total power drawn from the battery, through the resistors, increases. Similarly, by closing the switch, we add another resistor in parallel, so the total resistance of the circuit is smaller after the switch is closed. Accordingly, the circuit draws greater current, and so the power from the battery increases.

q	10	Perfect. The explanation must be clear, correct and unambiguous.
r	9	Correct, but the argument is mildly flawed or incomplete.
s	8	Correct, but the explanation is substantially ambiguous or incomplete.
t	6	Incorrect, but does show some knowledge of Ohm's law.
u	4	Argument contains some physics, but is for the most part non-sequitur.
w	2	All but blank.
x	0	Blank.

Question 3a) [15%]

The total resistance here is

$$R_{\text{total}} = R + \frac{1}{\frac{1}{2R} + \frac{1}{2R}} = 2R = 120\Omega$$

so that $I = 1$ Amp.

q	10	Calculation correctly gets $I = 1$ Amp by valid physics methods.
r	9	Minor arithmatic error
s	7.5	More substantial errors.
t	6.8	Indicates some understanding of Ohm's law, but resistance is calculated incorrectly or equivalent resistance is misinterpreted.
u	4	Some physics is involved, but may involve non-sensical calculations of R_{total}.
w	2	Irrelevant calculations
x	0	Blank

Question 3b) [10%]

In this circuit, both paths through the parallel element are identical. Therefore, Va = Vb. It is not correct to say simply that A,B are in parallel; a complete, correct answer must discuss the resistance of the entire circuit. It is not correct to discuss only the resistors in front of points A,B.

q	10	Perfect. An articulate, convincing argument discusses the resistance of the entire circuit.
r	9	Like q, but ambiguous or incomplete.
s	8.2	More or less correct, but ambiguous or incomplete.
t	6.5	Invokes Ohm's law but arrives at an incorrect answer.
u	4.5	Pays little mind to the real implications of Ohm's law. Argues that "the resistors go from parallel to series" or "one of the resistors is bypassed by the wire", etc. without mitigating arguments will end up in this category.
w	2	All but blank.
x	0	Blank.

Question 3c) [15%]

q	10	Perfect
r	9.6	Formula is correct but answer is wrong. If incorrect value from (a) was used, this is the highest score possible. Alternatively, correct answers following a somewhat incomplete or difficult to follow calculation may get this score.
s	8.0	Use of a correct formula but with wrong values for I or R, or calculation is difficult to follow.
t	6.5	Major errors leading to an incorrect solution. Using values for I, R or ΔV that cannot be accounted for may get this grade.
u	4	Some real physics, but not very coherent or relevant.
w	2	All but blank
z	0	Blank.

Question 3d) [5%]

Note that, as per (b), $V_a = V_b$. The two points are already at the same potential, so closing the switch does not change the circuit. Drawing an equivalent resistance diagram would be convincing here. In either case, the total resistance does not change, so the power drawn remains constant.

q	10	Perfect. THe power drawn from the battery does not change from the value in 3(c). For this category the student must clearly state or show algebraically that the two circuits are identical.
r	9.2	Minor errors or ambiguities.
s	8.0	Substaintally errors or ambiguity, but more less the right.
t	6.5	Incorrect, but at least based on an argument that involves real physics.
u	4.0	Incorrect and based nonsense. Solutions in which the new resistance is calculated, or declared, incorrectly with little or no argument receive this score. Solutions that, without justification, assign currents running through the closed switch or backward through the circuit, or "both ways" will probably receive this score.
w	2	All but blank.
x	0	Blank.

Question 3e) [10%]

Switch 2 shorts out the parallel resistors leaving only the one 60 ohm resistor. Alternatively, find the total resistance of the four resistors in parallel with a wire; it reduces to zero. A correct solution must demonstrate an understanding of Ohms law (aka, what happens to current when the resistance of the circuit drops). Correct solutions must indicate a correct relationship between current drawn and power dissipated (more current, more power; thats why short circuits start fires).

Q	10	Perfect. The solution shows that the new resistance is less than the initial resistance, so as per Ohms law, the current increases, so the total power supplied by the battery increases.
R	9	Like Q, but minor errors or ambiguities in the explanation.
S	7.5	Substantial errors or ambiguities, but more or less the right idea.
T	6.5	Incorrect, but mitigated by an argument that involves real physics.
U	3	Any answer that does not show a fundamental understanding of Ohms law or some familiarity with the concept of a short circuit receives this score. For the most part, solutions that indicate that resistance goes down and therefore so does power dissipated, or that say, "the switch is just a wire, so it does not dissipate power," or say, "there is only one resistor to dissipate the power, so the power goes down," are in this category.
W	2	All but blank or utterly nonsensical.
X	0	Blank.

Note: for question 4 you were marked down everytime you used "current is the same everywhere". The category **r** goes not apply for this mistake – it was continuously emphasised throughout the quarter!

Question 4a) [10%]

q	10	Stated that $I_A = I_B + I_C$, which implies that $A_a v_a = A_b v_b + A_c v_c$
r	7.5	Stated $I_a = I_b + I_c$ but could not re-write this equation in terms of speeds.
s	6.8	Stated that "current in = current out" but did not write an expression in terms of speeds.
t	6.5	Realised that current split at the junction and attempted to use the fluid model to come up with an (incorrect) expression.
u	3.0	Had all currents equal, or wrote $I = Av$ but no useful work.
x	0.0	Blank or effectively blank.

Question 4b) [10%]

q	10	Correctly stated that $\Delta P + \Delta\text{KE/vol} = 0$ (or equivalent).
r	8.8	1 error (e.g. included $\varepsilon_{\text{pump}}$, changes in PE, changes in KE). People who drew correct energy bubbles *without writing the corresponding equations* also fell into this category.
s	8.0	Had 2 errors of the type described in **r**.
t	6.0	Wrote the general fluid transport equation; did not figure out which terms were irrelevant.
u	2.0	Applied incorrect model (e.g. talked about the voltage change across the pipe) or severely mangled the fluid model.
w	1.0	Incomprehensible use of random fluids.
x	0.0	Blank or nothing appropriate

Question 4c) [10%]

q	10	Correctly stated that $\Delta P + \Delta\text{KE/Vol} = -I_c R$
r	8.8	1 error (e.g. included $\varepsilon_{\text{pump}}$, changes in PE, changes in KE). People who drew correct energy bubbles *without writing the corresponding equations* also fell into this category.
s	8.0	Had 2 errors of the type described in **r**.
t	6.0	Wrote the general fluid transport equation; did not figure out which terms were irrelevant.
u	2.0	Applied incorrect model (e.g. talked about the voltage change across the pipe) or severely mangled the fluid model.
w	1.0	Incomprehensible use of random fluids.
x	0.0	Blank or nothing appropriate

Question 4d) [10%]

q	10	Correctly got $I_A = A_a v_a = (0.01 \text{ m}^2/s)(6 \text{ m/s}) = 0.06 \text{ m}^3/s$.
r	9.2	Incorrect units, but had $I = Av$.
s	8.3	Used r=100 cm rather than $A_a = 100 \text{ cm}^2$.
t	7.5	Attempted to use the fluids model to find an (incorrect) value for I_a. Did not use $I = Av$ anywhere.
u	2.0	Used electrical quantities in an attempt to calculate I.
w	1.0	Said current and velocity were the same.
x	0.0	Blank or effectively blank.

Question 4e) [10%]

q	10	Correctly obtained $v_b = 7$ m/s. The easiest way of doing this was by using the fluid transport equation between points **a** and **b**.
a	10	Would be correct except for errors on the previous part.
r	7.9	Correct setup, minor math error
s	7.0	Correct setup, but major math error or final answer absent.
t	6.0	Used $I_a = I_b + I_c$, no clear explanation for where the v_b value came from.
u	5.0	Made some attempt to find v_b using pieces of the fluid model incorrectly, not clear what was being done.
w	2.0	Stated that $v_a = v_b = v_c = 6$ m/s, or that $I_a = I_b = I_c$.
x	0.0	Blank or nothing appropriate.

Question 4f) [10%]

q	10	Multiplied the answer from 4e) by A_b to get the current.
a	10	Would be correct except for errors on the previous part.
r	9.2	Minor error (e.g. no units, or units incorrect).
s	7.8	Correct setup, but final answer is wrong or absent.
u	4.0	Said that $I_a = I_b$ because current is constant.
w	2.0	Something is attempted, but direction or reasoning is unclear.
x	0.0	Blank or effectively blank.

Question 4g) [10%]

q	10	Correctly obtained $I_c = 0.0075$ m^3/s by using $I_a = I_b + I_c$.
a	10	Would be correct except for errors on the previous part.
r	9.2	Minor math errors, incorrect or missing units
s	7.8	Correct setup with no final answer given.
b	7.0	Tried using energy density equation with no resistance.
t	6.0	Tried to use $I = Av$ with no explanation of their value for v_c.
u	4.0	Said that $I_a = I_c$ because current is the same everywhere, or that $I_c = I_b$.
w	2.0	Something is attempted, but direction or reasoning is unclear.
x	0.0	Blank or effectively blank.

Question 4h) [10%]

q	10	Correctly obtained 3 m/s by $v_c = I_c/A_c$.
a	10	Would be correct except for errors on the previous part.
r	9.2	Minor math errors, incorrect or missing units
s	7.8	Correct setup with no final answer given.
b	7.0	Tried using energy density equation with no resistance.
u	6.0	Attempted qualitative explanation, no calculation.
w	4.0	Something is attempted, but direction or reasoning is unclear.
t	2.0	Said all speeds are the same, or all Is are the same.
x	0.0	Blank or effectively blank.

Question 4i) [10%]

q	10	Correctly stated the pipe with the highest current would fill the jug quickest.
a	10	Answer is only incorrect because of previous errors.
r	8.0	Answered pipe b or c because of higher velocity, not higher current.
s	6.8	Said that A is large, so $I = Av$ is large for pipe b.
u	6.0	Attempted qualitative explanation, no calculation.
t	3.0	Said that $I_b = I_c$ so it made no difference.
w	2.0	Answer without explanation or attempted an explanation without an answer.
x	0.0	Blank or essentially blank.

Question 4j) [10%]

q	10	Correct.
a	10	Answer is only incorrect because of previous errors.
r	9.2	Minor math error; error with units.
s	7.0	Physics error (e.g. used wrong current).
t	6.8	Correct setup, but no final answer given or an massively incorrect final answer.
u	6.0	Attempted to use the fluid model, but was unclear.
w	2.0	Tried to use electrical equations or another equally irrelevant model.
x	0.0	Blank or effectively blank.