The *definition* of the momentum $\mathbf{p}$ of a mass is

$$\mathbf{p} = m\mathbf{v}, \tag{1}$$

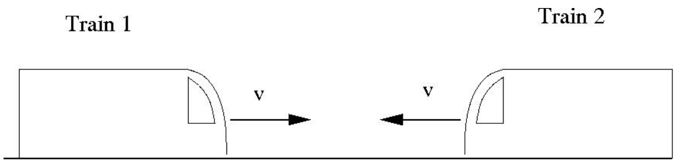
where m is the object's mass and $\mathbf{v}$ is its velocity. Notice that because the mass is a positive number that the vectors $\mathbf{p}$ and $\mathbf{v}$ are always parallel. The intuitive way to understand momentum is that it is "how hard something is to stop" or how much "oomph" it has. The more mass an object has or the faster it goes, the more momentum it has and the harder it is to stop.

We can also define the total momentum for a system, $\mathbf{p_{tot}}$, by adding all the momentum vectors for each part of the system. Most of the time we will be dealing with systems of two particles, for which

$$\mathbf{p_{tot}} = \mathbf{p_1} + \mathbf{p_2}$$

but this can easily be generalized to systems with any number of particles.

The idea of the momentum of a system can be a little bit confusing, however. Looking at the idea of momentum for a single particle, you may be tempted to say that the momentum of a system tells you "how hard the *system* is to stop". This is close, but misleading. Consider the following example: two trains of equal mass m are travelling at the same speed down a railway track in opposite directions. What is the total momentum of the system consisting of the two trains?



As the mass and the speed of the trains is the same, the length of the momentum vectors must be the same. As the velocity vectors are pointing in opposite directions, the momentum vectors must also point in opposite directions. Adding the two momentum vectors together gives us a $\mathbf{p_{tot}} = 0$! The total momentum refers to how hard the *center of mass* would be to stop, not the individual pieces of the system. We will learn more about the center of mass when we start to look at rotational motion in later DLs.

For the moment, forget about center of mass. The main reasons that the total momentum is of interest to us is

- The momentum is a state function – it only relies on what the mass and velocity is at that instant, not on how it got there.
- The total momentum of a closed system is conserved.

Here closed system means "no (net) push or pull is exerted on the system". This is a different concept from a closed *energy* system, which meant "no heat is exchanged with the environment, nor is any external work done on or by the system." It is possible to find open momentum systems which are closed energy systems, and it is possible to find open energy systems which are closed momentum systems.

Let us go back to our example of the two trains, and let us imagine that they collide.
Q: If train 1 comes to rest after the collision, what is the velocity of train 2?
By conservation of momentum, the total momentum before the collision is the same as the total momentum after the collision. From the work we did above, the total momentum before was zero. As train 1 comes to rest, we know $\mathbf{p_{1f}} = 0$. Therefore

$$\mathbf{0} + \mathbf{p_{2f}} = \mathbf{0}$$

This means that $\mathbf{p_{2f}} = m\mathbf{v_{2f}} = 0$. Because the mass of the train is not zero, we are forced to conclude that it comes to rest as well.

This was a simple example, but for complicated examples we use a momentum chart. For a closed system we know $\mathbf{p_{tot,i}} = \mathbf{p_{tot,f}}$, so that $\mathbf{\Delta p_{tot}} = 0$. We put this in a chart in the following way:

	$\mathbf{p_i}$	+	$\mathbf{\Delta p}$	=	$\mathbf{p_f}$
Train 1	$\longrightarrow$	+	???	=	$\mathbf{0}$
Train 2	$\longleftarrow$	+	????	=	????
Total	????	+	$\mathbf{0}$	=	????

Here we have filled in the initial momenta, as we know what they are. We have also used the fact $\mathbf{\Delta p_{tot}} = 0$ on the bottom line. I have also used the fact that I know train 1 comes to rest after the collision. Note that the arrows are to scale! By adding the initial momenta of train 1 and train 2 we find that the initial total momentum is zero and enter this into the chart as well:

	$\mathbf{p_i}$	+	$\mathbf{\Delta p}$	=	$\mathbf{p_f}$
Train 1	$\longrightarrow$	+	????	=	$\mathbf{0}$
Train 2	$\longleftarrow$	+	????	=	????
Total	$\mathbf{0}$	+	$\mathbf{0}$	=	$\mathbf{0}$

Looking at the last column of this table we see that $\mathbf{0} + \mathbf{p_{2,f}} = \mathbf{0}$, implying that $\mathbf{p_{2f}} = \mathbf{0}$ as claimed. By doing vector subtraction in rows 1 & 2 we can complete the chart to give

	$\mathbf{p_i}$	+	$\mathbf{\Delta p}$	=	$\mathbf{p_f}$
Train 1	$\longrightarrow$	+	$\longleftarrow$	=	$\mathbf{0}$
Train 2	$\longleftarrow$	+	$\longrightarrow$	=	$\mathbf{0}$
Total	$\mathbf{0}$	+	$\mathbf{0}$	=	$\mathbf{0}$

Note that conservation of momentum cannot tell us what will happen if the two trains collide; it only told us that if the two trains collide *and train 1 is stopped* then train 2 will necessarily stop as well. Conservation of momentum would also hold if both trains bounced back at half their initial speed. See if you can work your way through the momentum chart to show this is a solution.

Conservation laws are good at telling us what cannot happen.
They generally cannot tell us what will happen (unless there is only one unknown).

(You may be confused by the fact that the Earth is pulling down on the trains, yet we have not included it in our "closed momentum system". While it is true that the Earth's gravity pulls the trains down (making them heavy) the ground also pushes them up, and that these two cancel each other exactly. We will be able to be more precise when we discuss *forces*.)