The tablecloth "trick"

Damien Martin

April 18, 2006

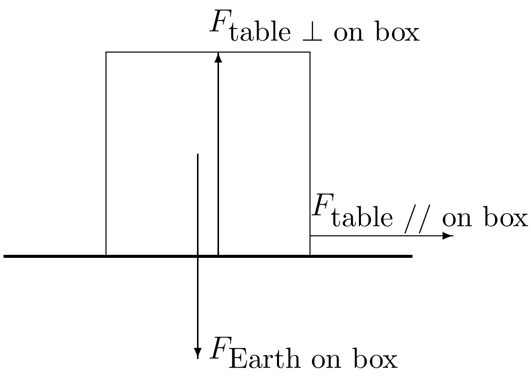
It has become a routine in physics 7B to explain the tablecloth trick in terms of impulse. I outline what I believe to is being looked for in an explanation, and point out why I think that the wrong physics is being used to get there. But first, what is the tablecloth trick?

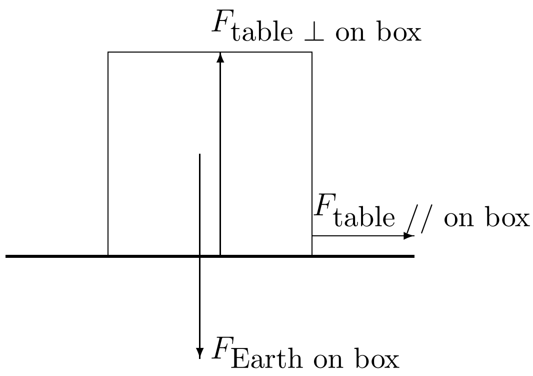
> The *tablecloth trick* is the phenomenon of pulling a tablecloth quickly from underneath an object, and so it remains (almost) unmoved. In contrast, if the tablecloth is moved slowly then the object gets dragged along indefinitely. What we are required to explain is why pulling at different speeds makes such a large difference

1 The forces acting

Regardless of how you wish to explain the tablecloth trick, there are some elements that are going to remain the same. In particular, any approach is going to require us to look at forces on the object. As I only want to deal with one coefficient of friction, let us look at the initial instant where we start to pull the boxes:

Fast pulling **Slow pulling**





The only difference between these two diagrams is that "friction" (i.e $F_{\text{table // on box}}$) is *smaller* on the slowly moving box. It seems somewhat paradoxical as the box with more force on it is the one that cannot keep up! This is easy enough to understand, however:

$$F_{\text{table // on box}} \leq \mu F_{\text{table} \perp \text{on box}} = \mu m_{\text{block}} g$$

where I have used balance of forces in the vertical direction and the standard expression for the force of gravity.[1]

2 In terms of impulse

A look at this problem from the point of view of impulse would require something along these lines:

1. For the object to get to a greater speed, we need $m_{\text{block}} v_f$ to be big.
2. As the object started from rest and $J = \Delta p$, we have $J = m_{\text{block}} v_f - 0 = m_{\text{block}} v_f$.
3. We can use the relationship $J = (\sum F)_{\text{ave}} \Delta t$ to get an idea of what is going on
   - If the object slides off, then Δt is small. This tells us that J is small (as $\sum F$ has a maximum value). Therefore v_f is also small.
   - if the object stays in contact, then Δt is large. The force does not *vanish* in this case, so v_f is going to be large and the object can stay on the paper.

The above reasoning looks acceptable, so why am I complaining about it? One problem is that we have to ask the question "what are we trying to explain?". If we are trying to explain why the object stays on the paper, then we cannot put Δt in "by hand". The net force only acts while it is on the paper: if we say the contact time is small we are implicitly saying that it will fall off the paper! As an explanation, it is completely circular.

[1] At first, this seems against what is said in the notes for this activity. The claim is that friction is *always* μmg. This cannot be true, otherwise objects could not sit on a table at rest as friction would be accelerating it. The equation above, at least for objects that are not moving, is correct.

While by itself, this would make the problem bad the problems do not stop here. Another objection is to do with the force. In the first point, I can take it as experimental that the frictional force saturates at some point, otherwise I could never push anything without an infinite force! The second point is tricker – I claim that the frictional force does not vanish. A good question is how do I know this? The answer is that the frictional force will become as big as it needs to be to keep the object on the paper, *up to its maximum value.* Again, I am presupposing that the object stays on the paper to explain why it stays on the paper!

Finally, there is the issue of which states should be the initial and final states. I would argue that this becomes an issue because we use an inappropriate description of the problem.

3 Analysis using forces

So how do we show that the object stays on the paper sometimes and falls off other times? Consider two cars starting at a red light. As the light turns green, the two cars stay abreast all the time if (and only if) they are both going at the same velocity. If one gets faster, so does the other one. In particular, the *accelerations* of these two cars must stay the same.

But what does this tell us about the block and the paper? Well, in order for them to remain in contact without slipping, we require the block to have the same velocity as the paper at all times. Just like the cars above, if one speeds up the other must also speed up. The *accelerations* of the block and the paper must be identical.

Let us say that we are accelerating the paper with an acceleration a_{paper}. For the block not to slip off, we must have $a_{\text{block}} = a_{\text{paper}}$. This is also a condition on the net force on the block:

$$\sum F_{\cdot onblock} = m_{\text{block}} a_{\text{block}} \tag{1}$$

This is just Newton's second law, and would apply in any situation. Using the force diagram, we see that

$$\sum F_{\cdot onblock} = F_{\text{table // on block}} \leq \mu m_{\text{block}} g. \tag{2}$$

The acceleration of the block is given by equating these two and rearranging:

$$m_{\text{block}}a_{\text{block}} = F_{\text{table // on block}} \Rightarrow a_{\text{block}} = \frac{F_{\text{table // on block}}}{m_{\text{block}}} \quad (3)$$

But as $F_{\text{table // on block}}$ has a maximum, we can tell that

$$a_{\text{block}} \leq \frac{\mu m_{\text{block}} g}{m_{\text{block}}} = \mu g \quad (4)$$

Thus, if I pull the paper with an acceleration $a_{\text{paper}} > \mu g$, then the block cannot possibly keep up. [2] I suppose one could then go and calculate the impulse, but it would no longer serve any purpose.

4 A compromise analysis

So is there anyway of looking at this problem that deals with impulse and is non-circular? The answer is "yes", although I believe that the analysis in terms of forces is the superior way of looking at the problem. It is absolutely necessary to state what (static) friction does correctly:

$$F_{\text{table // on box}} \leq \mu m_{\text{block}} g. \quad (5)$$

The argument would then go

1. There is a maximum force that friction can deliver.

2. For the object not to slip, it must have the same velocity as the paper. Therefore, if I change the velocity of the paper by Δv in an interval Δt, then the block must undergo the same Δv in the same Δt.

3. The impulse imparted to the block is then $J = m_{\text{block}}\Delta v$.

4. The force needed to impart this is

$$(\sum F)_{ave} = \frac{J}{\Delta t} = m\frac{\Delta v}{\Delta t} \quad (6)$$

[2] Technical note: once this occurs, the block starts to move and strictly speaking we would have to use the coefficient of *kinetic friction* μ_k, rather that μ to calculate the frictional force. This does not change the conclusion as $\mu_k < \mu$, (corresponding to the fact it is easier to push an object once it is moving than to get it to move in the first place) and so it creates even less frictional force.

5. By looking at the force diagram, and using the fact that we know the frictional force has a maximum value, we see that

   - if Δv is big enough or Δt is small enough that the net force cannot accelerate the block. It falls off.
   - if Δv is small enough or Δt is long enough that the net force can accelerate the block and it stays on.

For me, this gets into the uncomfortable regime of discussing Δt, which leads back to the circular part of the argument. We have managed to include impulse as well, but it is in there in a rather artificial way as I have used it as a route to getting to Newton's second law instead of just using it directly. However, I feel that this explanation is better than the first explanation as it shows that it is the bounded nature of friction responsible for this trick.