FNT #8
A look at polarisation

August 22, 2005

Question 1 – the fence-post model

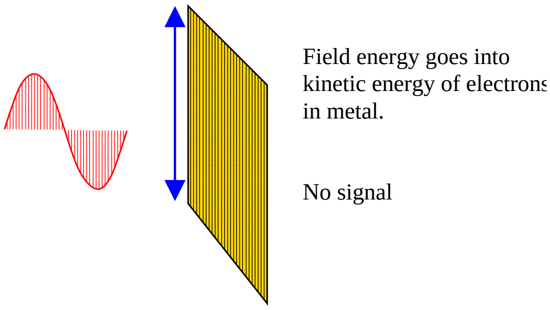
Consider transverse waves on a horizontal rope that is strung between vertical fence posts.

a) Which polarisations will pass through the fence posts? Which will not?

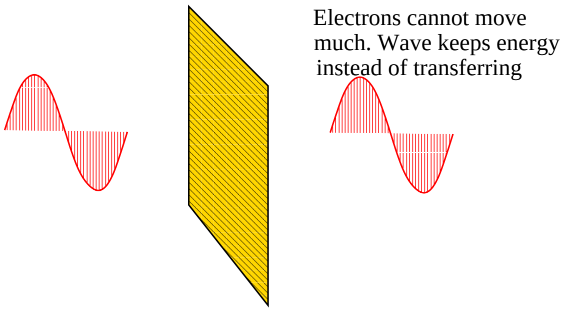
Rope that tries to move to the sides will hit the posts. So only rope travelling up and down (or that component) will get through.

b) Why is this a bad model for light polarisation?

Let us treat the metal as the fence-posts. Then if the light comes along we have



But this is the light that the "fence-post model" would suggest would get through. If I keep the light the same, but turn the polariser around instead I have



Electrons cannot move
much. Wave keeps energy
instead of transferring

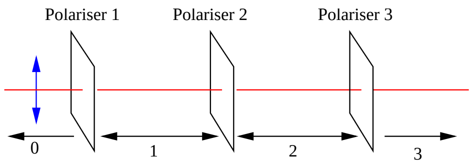


Figure 1: The setup of the polarisers. We divide the problem into four regions (0,1,2 and 3).

Question 2 – Combinations of polarisers

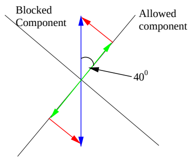
We have a setup of three polarisers as shown in figure 1. In each question we change the orientation of the polarisers slightly to see what gets through. I am going to work through parts (a) and (b) throughly, then do the general case and apply the formula for parts (c) and (d).

Important!
The orientation of the polariser refers to which field can get through, NOT the direction of the material in the polariser

a) Polariser 1: $0°$, Polariser 2: $40°$, Polariser 3: $75°$

Note that all angles are given with respect to the vertical. To start with we have vertically polarised light going through a vertical polariser. Not suprisingly, all the light gets through. So in region 1 we have all the light, and it is still vertically polarised.

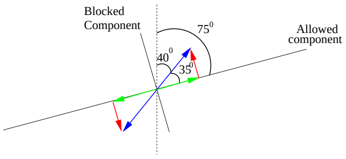
The second polariser is at $40°$ to the vertical. We need to break the polarisation vector into components along this direction and its normal



The green (component of the) electric field goes through while the red (component of the) electric field is blocked. So in region 2 we have light polarised at $40°$ to the vertical with an amplitude

$$E_2 = E_0 \cos 40°$$

Going from region 2 to region 3 we have light polarised at 40° to the vertical passing through a polariser at 75° to the vertical. Diagram-wise this is



So we can relate the final magnitude to the initial by using trigonometry. We have

$$\begin{aligned} E_3 &= E_2 \cos 35^\circ \\ &= (E_0 \cos 40^\circ) \cos 35^\circ \\ &= E_0 \cos 40^\circ \cos 35^\circ \\ &\approx 0.6275 E_0 \end{aligned}$$

But we are interested in the intensity at the other end, which is related to the *square* of the amplitude by some constant. We have

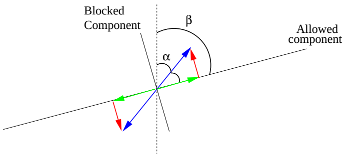
$$\begin{aligned} I_3 &= kE_3^2 \\ &= (0.6275)^2 (k(E_0)^2) \\ &= 0.3938(kE_0^2) \\ &= 0.3938 I_0 \end{aligned}$$

We were told that $I_0 = 1550$ W/m^2 in the beginning of the question. So the final intensity is

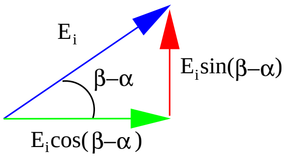
$$I_3 = 610\,\mathrm{W/m^2}$$

A general rule

When going through a polariser, we are trying to find the component of the electric field along the direction of the polariser. Let us look at the light before at an angle α, and a polariser at angle β.



Let us take that central triangle and use trig. to find the components. We know the inner angle is $\beta - \alpha$ from the figure. Redrawing the triangle we have



Here we see the amplitude of the next part of the field E_{i+1} is related to E_i by

$$E_{i+1} = E_i \cos(\beta - \alpha).$$

Let us relate this more directly to the question we have. We start with light polarised at 0°, passing through a polariser at an angle θ_1. So we have

$$E_1 = E_0 \cos\theta_1.$$

The light that exits the first polariser is polarised at an angle θ_1. So when it goes through the second polariser (at an angle of θ_2 to the vertical) we have

$$E_2 = E_1 \cos(\theta_2 - \theta_1).$$

We can relate this back to the amplitude of the original electric field by using the relationship above.

$$E_2 = (E_0 \cos\theta_1)\cos(\theta_2 - \theta_1).$$

The light is now polarised along the direction θ_2, and it should be easy to show that

$$E_3 = E_0 \cos\theta_1 \cos(\theta_2 - \theta_1)\cos(\theta_3 - \theta_2)$$

To relate all of this back to intensities we have

$$I_3 = I_0 \cos^2\theta_1 \cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$$

b) $\theta_1 = 30°$, $\theta_2 = 30°$, $\theta_3 = 70°$

Here we have

$$\begin{aligned} I_3 &= I_0 \cos^2 30° \cos^2(30° - 30°)\cos^2(70° - 30°) \\ &= I_0(0.75)(1)(0.5868) \\ &= 0.44 I_0 \end{aligned}$$

which gives

$$I_3 = 0.44 \times 1550\,\mathrm{W/m^2} = 682\,\mathrm{W/m^2}$$

Sometimes some intuition gets lost when plugging into long formulas. Note that the second polariser did *nothing.* The light was polarised at 30° from the first polariser, so all the light passed through the second polariser.

c) $\theta_1 = 0°$, $\theta_2 = 45°$, $\theta_3 = 90°$

This is pretty simple:

$$\begin{aligned} I_3 &= I_0 \cos^2(0°) \cos^2(45° - 0°) \cos^2(90° - 45°) \\ &= I_0(1)(0.5)(0.5) \\ &= 0.25 I_0 \\ &= 0.25 \times 1550\,\mathrm{W/m^2} = 387.5\,\mathrm{W/m^2} \end{aligned}$$

d) $\theta_1 = 0°$, $\theta_2 = 90°$, $\theta_3 = 45°$

This should be zero – after all we go through one polariser that makes the light vertical and then another that takes the horizontal component. Surely enough

$$\begin{aligned} I_3 &= I_0 \cos^2 0° \cos^2(90° - 0°) \cos^2(45° - 90°) \\ &= I_0(1)(0)(0.5) \\ &= 0 \end{aligned}$$

Question 3 – A lens question

A near sighted person wears contacts to correct for a far point that is 3.62 metres from his eyes. The near point of his unaided eyes us 25.0 cm from his eyes. If he does not remove his lenses when reading, how close can he hold the book and still read clearly?

Answer:

Let us recall what we know. The near point is fine at 25 cm, and so we are not getting the lens to correct for that. Instead the lens should make an image of an object a long way away appear to be approximately 3.62 metres from the eye. Then the eye is capable of focusing on the image created by the lens.

As this is a *contact* lens, we know that the distance from the eye is the same as the distance from the lens. We are trying to correct for objects near infinity:

- $o_{\text{contact}} = \infty$
- $o_{\text{eye}} = 3.62$ m, so that the eye can focus.
- $i_{\text{contact}} = -o_{\text{eye}} = -3.62$ m.

Using the thin lens equation for the contact lenses we find

$$\begin{aligned} \frac{1}{o_{\text{contact}}} + \frac{1}{i_{\text{contact}}} &= \frac{1}{f_{\text{contact}}} \\ 0 - \frac{1}{3.62\,\text{m}} &= \frac{1}{f_{\text{contact}}} \end{aligned}$$

So we conclude

$$f_{\text{contact}} = -3.62\,\text{m}$$

We now want to know how close we can bring an object to the eye. The near point is 25 cm, so the object *for the eye* cannot be any closer than this. Remember that the object *for the eye* is the image created by the contact lens. See figure 2 for an idea of the layout. So we want

$$i_{\text{contact}} = -25\,\text{cm}$$

so that the image is 25 cm behind the contact lens (and hence 25 cm in front of the eye, at its near point).

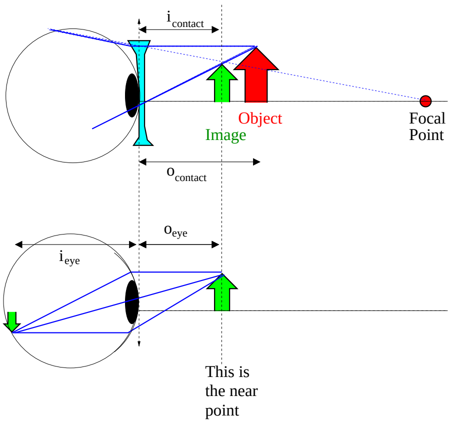


Figure 2: In the upper diagram we see the contact lens focusing an object to a (virtual) image. To find what happens when the eye looks at it we go to the second picture, where we treat the image as the object *for the eye* and ignore the contact lens. The eye can still focus on the green arrow as it is located at the near point.

We apply the thin lens equation to the contact lens again

$$\begin{aligned}
\frac{1}{o_{\text{contact}}} + \frac{1}{i_{\text{contact}}} &= \frac{1}{f_{\text{contact}}} \\
\frac{1}{o_{\text{contact}}} - \frac{1}{0.25\,\text{m}} &= -\frac{1}{3.62\,\text{m}} \\
\frac{1}{o_{\text{contact}}} &= -\frac{1}{3.62\,\text{m}} + \frac{1}{0.25\,\text{m}} = 3.72376\,\text{m}^{-1}
\end{aligned}$$

This gives us an object distance of

$$o_{\text{contact}} = 26.85\,\text{cm}$$

As expected from figure 2, this is further away than if the person had taken off their glasses to start with.