FNT #5
Refraction

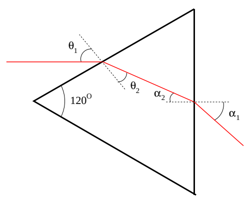
August 16, 2005

There is only one problem in this set of FNTs, the prism problem: Consider the prism shown below, surrounded by a clear oil. Light travels through the oil 2.5 times faster than it does through the prism material. This prism "lens" is illuminated by a light ray as shown (perpendicular to the plane of the "back" surface of the prism).

a) What is the angle the ray makes with the horizontal when it exits the prism "lens"?

To answer this, let us first get a good picture of what we want. We know that the light travels *faster* in oil, so the oil has a *lower* refractive index. This means when we go from oil to the prism we bend *toward* the normal, and as we go from the prism to the oil we bend *away* from the normal.

(These statements will all be backed up by mathematics and the working that we are going to do. I give these arguments now so that we can draw a picture that will look something like the end result)



The question is asking us to find the value of α_2.

Let us ask what we know about the refractive indices. We know

$$n_{\text{oil}} = \frac{c}{v_{\text{oil}}}$$

and that $v_{\text{oil}} = 2.5 v_{\text{prism}}$. Therefore we have

$$n_{\text{oil}} = \frac{c}{2.5 v_{\text{prism}}} = \frac{1}{2.5}\frac{c}{v_{\text{prism}}} = \frac{1}{2.5} n_{\text{prism}}$$

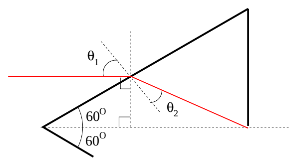
or that

$$n_{\text{prism}} = 2.5 n_{\text{oil}}.$$

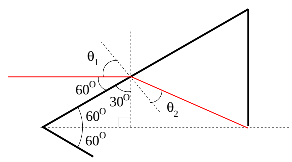
This is simply the mathematical version of exactly the same reasoning given at the beginning of the problem.

Some trig

Now we need to do some trigonometry to figure out the angles. Let us focus on getting θ_1 first. We draw a small right angle triangle and mark the right angles:



The sum of all the angles in the triangle are 180 degrees, so this tells us the the upper angle is 30 degrees. We then use the other right angle, and the angle of the upper corner of the triangle to find that it is sixty degrees:



Using the fact that the normal (the dotted line) is at 90 degrees to the prism's edge, we find that $\theta_1 = 30°$.

Snell's law

Finding θ_2 is a case of using Snell's law. We have

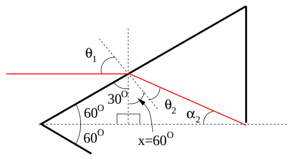
$$n_{\text{oil}} \sin\theta_1 = n_{\text{prism}} \sin\theta_2$$

Using what we know about n_{prism} we have

$$\begin{aligned} \cancel{n_{\text{oil}}} \sin 30^\circ &= (2.5\cancel{n_{\text{oil}}}) \sin \theta_2 \\ 0.5 &= 2.5 \sin \theta_2 \\ \sin \theta_2 &= 0.2 \\ \theta_2 &= 11.53^\circ \end{aligned}$$

More trig

Now we have to find how α_2 and θ_2 are related.



We know that the 30 degree angle and the angle x together make up a right angle. This tells us

$$30^\circ + x = 90^\circ \Rightarrow x = 60^\circ.$$

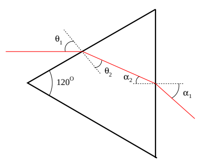
We also know that the sum of the three angles in a right angled triangle are 180°. This tells us

$$\begin{aligned} 90^\circ + (60^\circ + \theta_2) + \alpha_2 &= 180^\circ \\ \Rightarrow \alpha_2 &= 30^\circ - \theta_2 \end{aligned}$$

Using our value of θ_2 we get $\alpha_2 = 18.46^\circ$.

Snell's law again

Now we focus on the back region of the prism. The appropriate picture is



As α_1 and α_2 are the angles from the normal, we can use them in Snell's law:

$$n_{\text{prism}} \sin \alpha_2 = n_{\text{oil}} \sin \alpha_1$$

Again, we work through to get

$$\begin{aligned}(2.5\cancel{n_{\text{oil}}})\sin\alpha_2 &= \cancel{n_{\text{oil}}}\sin\alpha_1 \\ \sin\alpha_1 &= 2.5\sin\alpha_2 \\ &= 2.5\sin 18.46^\circ = (2.5)(0.3167) = 0.7917\end{aligned}$$

So we get

$$\alpha_1 = \sin^{-1} 0.7917 = 52.34^\circ$$

b) What would happen if we replaced the oil with air?

Before we had

$$\frac{n_{\text{prism}}}{n_{\text{oil}}} = 2.5$$

When we replace oil with air, we have

$$\frac{n_{\text{prism}}}{n_{\text{air}}} > 2.5$$

as the speed of light in air is (almost) exactly the same as c, the speed of light in vacuum.

So whenever we have refraction, the bending is going to be more extreme. The angle θ_2 is bent toward the normal, so θ_2^{air} is going to be *smaller* than θ_2^{oil}. We then have

$$\alpha_2 = 30^\circ - \theta_2$$

so if θ_2^{air} is smaller, α_2^{air} will be bigger.

Finally, we have refraction again. It is at a greater incident angle α_2, which will make the refracted angle bigger α_1. We have an additional affect: the ratio of refractive indices is bigger, so the bending will be more extreme increasing α_1 even further. So we conclude

$$\alpha_1^{\text{air}} > 52.34^\circ$$