FNT #9: Electric fields and voltages

Damien Martin

November 12, 2005

Question 1: Electric field and electric voltage

The electric field E and the voltage V at a point $r = R$ are related by

$$E(r = R) = \left|\frac{\Delta V}{\Delta R}\right|$$

This tells us that the electric field is related to how quickly the voltage changes. Here the "Δ" means we are looking at finite differences in the voltage and distance, and doing a rise over run. A more elegant and precise way of doing it is to look at the tangent at the same point:

$$E(r = R) = \left|\frac{\mathrm{d}V}{\mathrm{d}r}\right|$$

Written in this form, we are reminded that the functional form of E can be obtained from the functional form of V via a derivative.

Given that the field of a point change E is given by

$$E = \frac{k|Q|}{r^3}$$

find the correct expression for the Voltage from a charge.

Solution

This question is a little bit tricky. To start with, we have that the electric field is the negative slope of the electric field:

$$E = -\frac{\mathrm{d}V}{\mathrm{d}r}.$$

This should be given in the problem, but it is in your notes.

The rest of this problem is realising that to "undo" a derivative we need to integrate. That is

$$V = \int \frac{\mathrm{d}V}{\mathrm{d}r}\,\mathrm{d}r.$$

This is the *fundamental theorem of calculus.* If you have trouble remembering it, the easiest way is to think of the dr on the "denominator" and the dr in the "numerator" cancelling:

$$\int \frac{\mathrm{d}V}{\cancel{\mathrm{d}r}}\,\cancel{\mathrm{d}r} = \int \mathrm{d}V = V.$$

To finish the problem, we need to actually do the integral:

$$\begin{aligned} V &= \int \frac{\mathrm{d}V}{\mathrm{d}r}\,\mathrm{d}r \\ &= \int (-E)\,\mathrm{d}r \\ &= -kQ\int \frac{1}{r^2}\,\mathrm{d}r \\ &= +\frac{kQ}{r} \end{aligned}$$

Question 2: An α particle

An α particle is a charged particle emitted by many radioactive elements (e..g uranium-238, the most commen isotope of uranium) during radioactive decay. Composed of two protons and two neutrons, this form of nuclear radiation has little penetrating power and is only harmful if taken internally, or if it enters through a would or the eyes.

Some useful information:

Particle	Mass	Charge
Proton	$m_p = 1.602 \times 10^{-27}$ kg	$q_p = 1.602 \times 10^{-19}$ C
Neutron	$m_n = 1.673 \times 10^{-27}$ kg	$q_n = 0$ C

a) Calculate the electric potential due to an α particle at the following locations: i)$r_1 = 1.44 \times 10^{-9}$ m and ii) $r_2 = 2.88 \times 10^{-9}$ m

This is just a case of plugging in values into the formula. We are looking at the voltage in the field created by an α particle at a distance r:

$$V = \frac{kQ_\alpha}{r},$$

as we found in question 1. To work out the charge of the α particle, we note that it is made up of two neutrons and two protons:

$$\begin{aligned} Q_\alpha &= 2q_{\text{proton}} + 2q_{\text{neutron}} \\ &= 2(1.602 \times 10^{-19}\,\text{C}) + 2(0) \\ &= 3.204 \times 10^{-19}\,\text{C} \approx 3.2 \times 10^{-19}\,\text{C}. \end{aligned}$$

I am using the approximate value, as the precision given in this problem is not warrented (see my comment later on the mass). You may keep the full precision, but I am not going to!

At a distance $r_1 = 1.44 \times 10^{-9}$ C from the α particle the voltage is

$$\begin{aligned} V(r = 1.44 \times 10^{-9}\,\text{m}) &= \frac{(9 \times 10^9\,\text{N m/C}^2)(3.2 \times 10^{-19}\,\text{C})}{1.44 \times 10^{-9}\,\text{m}} \\ &= 2.0\,\text{N/C} = 2.0\,\text{V} \end{aligned}$$

We could perform the same calculation for r_2, but it is easier to note that as the distance has *doubled* that the voltage has to *halve*:

$$V(r = 2.88 \times 10^{-9}\,\text{m}) = 1.0\,\text{V}$$

b)

Another α particle is placed at r_1 so that it is not moving, while the first α particle is *held at rest*. Does the second α particle move through $r_3 = 0.77 \times 10^{-9}$ m or $r_2 = 2.88 \times 10^{-9}$ m?

Solution:

There are three separate ways of getting the right answer:

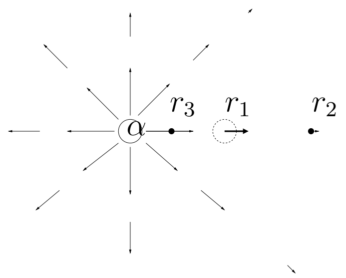
1. The easiest way is to think "like charges repel". As both the α particles are positively charged they will repel. Therefore the α particles are going to move apart. So the second α particle is going to move further away – to r_2.

2. A better way, which uses the physics that we have learnt so far is to think about the electric field. The first α particle is a positive charge, so it sets up an electric field pointing away from it. The second α particle is also positive, so it will feel a force in the direction of the electric field. Thus the two α particles will move apart, so the second α particle will travel through r_2.

3. The potential energy of a positive charge decreases as you move along the electric field. For the particle to begin moving, it needs to increase kinetic energy. That energy needs to come from somewhere, and it comes from the potential energy stored in the field. Therefore the particle has to go in the direction of the electric field, so the second α particle goes further away. So the answer is r_2.

The third of these explanations is by far the hardest, although it uses very little more than the conservation of energy and knowing which way potential energy increases and decreases.

The first and the second answer seem to be essentially the same. That is because they *are*. The first answer is the "direct method", which we consider the force between two particles directly. In the second answer, we let the first charge set up a field and then look at how this field influences the second particle. While the direct method is easier, I would like you to try and think about it in the field method. The reason that I want you to think about it this way is that we often do not know what the charges are, and we only know about the field they produce. This is certainly the case for the earth, which has an electric and magnetic field that is difficult to calculate but easy to measure. Instead I am trying to get you to think about fields while still in a familiar setting so you can check you answer using the "direct method". So much so, that I want to present it as a picture:



The dotted arrow shows where the second α particle is placed. The only arrow that is important is the **bold** one, as it is the only one the second α particle sees. Following it, we see we are driven to r_2, not r_3.

c)

Find the speed that the second α particle has as it passes through r_2.

Solution:

We know that the change in *total* energy is zero, by the conservation of energy. We have

$$\Delta\text{PE} + \Delta\text{Kinetic} = 0$$

We start by looking at the change in potential energy. We know

$$\Delta\text{PE} = q_\alpha \, \Delta V.$$

The change in voltage we calculated back in part a)

$$\begin{aligned}\Delta V &= V_f - V_i \\ &= (1.0\,\text{V}) - (2.0\,\text{V}) \\ &= -1.0\,\text{V}\end{aligned}$$

This allows us to calculate the change in potential energy:

$$\Delta\text{PE} = (3.2 \times 10^{-19}\,\text{C})(-1.0\,\text{V}) = -3.2 \times 10^{-19}\,\text{J}$$

Because this is the amount that potential energy has *decreased* by, we know that the kinetic energy must have increased by the same amount. Because the kinetic energy was initially zero, we have

$$\frac{1}{2} m_\alpha v^2 = -\Delta\,\text{PE} = +3.2 \times 10^{-19}\,\text{J}$$

wher v is the velocity at $r = r_2$. We can solve this equation by looking up the α particle mass: $m_\alpha = 6.55 \times 10^{-27}$ kg. The final velocity is then

$$\begin{aligned}v &= \sqrt{\frac{2(+3.2 \times 10^{-19}\,\text{J})}{m_\alpha}} \\ &= \sqrt{\frac{(2)(+3.2 \times 10^{-19}\,\text{J})}{6.55 \times 10^{-27}\,\text{kg}}} \\ &= 9885\,\text{m/s}\end{aligned}$$

The reason that we had to look up the α particle mass and *not* just add the masses of the two protons and neutrons we learnt in physics 7A. Some of that mass gets converted into energy when we bang all these particles together. This energy is what comes out of the sun – it is not altogether negligible! Using the wrong mass leads to an error of about 1% in the final answer.