FNT #8: The Cartman problem

Damien Martin

November 2, 2005

Question 1

Cartman is having trouble watching "The Terrance and Phillip Show" because of his extreme near-sightedness. His doctor advises that he switch from contact lenses to eyeglasses. If his current prescription is $f^{-1} = -6.50$ Dipoters, what prescription will he need for his glasses? To answer this you need to know that Cartman's glasses sit 1 cm in front of his eyes.

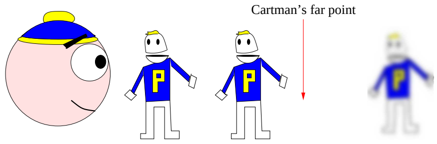
Solution:

First, let us remind ourselves of how multiple lens systems work. Basically it is a two-step process:

1. Using the object, find the image created by the *first* lens.

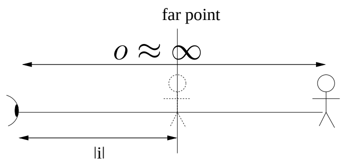
2. The second lens cannot tell the difference between the original object with a lens, and the image of the first lens by itself (by definition of what an image *is*). So we can ignore the first lens and the original object, and just pretend that the *first image* is the object for the second lens.

We define the *far point* d_{fp} to be the furthest distance away that someone can still see something clearly. So if Cartman was not wearing any eyeglasses, he can only see things closer to him than d_{fp}:



The copy of Phillip that is further away than Cartman's far point looks very blurry.

The whole point of corrective lenses is to take the light from Phillip far away and create an image somewhere that Cartman can see. Very far away objects should create an image at the far point (you could make it closer than this as well, but that creates problems for him looking at things that are close. The safest thing to do is give the "weakest possible lenses"). So using Cartman's contacts, we can find the far point:



We can use the thin lens equation to find the image distance. First, we use the fact that we know the focal length of the contacts:

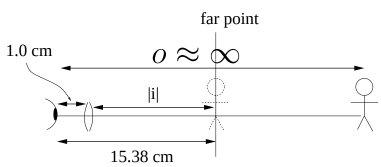
$$f_{\text{contact}} = \frac{1}{D_{\text{contact}}} = -\frac{1}{6.5\,\text{D}} = -15.38\,\text{cm}$$

Plugging this into the thin lens equation, we have

$$\begin{aligned} \frac{1}{o} + \frac{1}{i} &= \frac{1}{f_{\text{contact}}} \\ 0 + \frac{1}{i} &= \frac{1}{f_{\text{contact}}} \\ \Rightarrow i = f_{\text{contact}} &= -15.38\,\text{cm} \end{aligned}$$

The negative sign simply tells us that we are creating a virtual image. Cartman's far point is 15.45 cm from his face.

Now we move onto the glasses. The glasses job is still to create an image where Cartman can see it. Cartman's eyes have not changed, so his far point is in the same place. However, now the lenses (the glasses) are a centimetre closer to the far point. Since the image does not move, the image distance *as measured from the lens* gets 1 cm closer to zero as shown below. Remember this is a virtual image so $i < 0$:



Now that we know o and i, we can find the focal length of the glasses from the thin lens equation:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f_{\text{glasses}}}$$
$$0 + \frac{1}{i} = \frac{1}{f_{\text{glasses}}} \Rightarrow f_{\text{glasses}} = i = -14.38\,\text{cm} = -0.1438\,\text{m}$$

The prescription is then

$$D_{\text{glasses}} = \frac{1}{f_{\text{glasses}}} = -6.95\,\text{Diopters}$$