Question 1

Two 1-D waves of amplitude A and period T and constant phase ϕ are superposed. If wave 1 travels 13.5 wavelengths further than wave 2, will the interference be constructive, destructive or partial?

When you answer quiz questions, we look very critically at your answers. So it is only fair that our questions come under the same scrutiny. The question says that wave 1 travels 13.5 wavelengths further than wave 2. Further until what? It just stops travelling? It becomes a 2-D wave instead?

Of course we know what is being asked, I just want to point out that clarity is important and we will pick you up on it (so we should apologise when we do it ourselves!). It means that we have picked a particular point, and we are interested in the interference at that particular point. Wave 1 travels 13.5 wavelengths further than wave 2 to get to that point. What type of interference is at that point?

The quick answer is that everything about these waves is the same except the path length difference. The path length differs by 13.5 wavelengths. Shifting it over 13.5 wavelengths leads to completely destructive interference.

A slightly longer answer is that the path difference is 13.5 wavelengths. This corresponds to a total phase difference of

$$\begin{aligned}\Delta\Phi &= \frac{2\pi}{\lambda}(x_1 - x_2) = \frac{2\pi}{\lambda}([x_2 + 13.5\lambda] - x_2) \\ &= \frac{2\pi}{\lambda}(13.5\lambda) = 27\pi.\end{aligned}$$

As an *odd* multiple of π, we know that the interference is destructive.

Question 2

Two 1-D waves of amplitude A and period T are superposed. The difference between their fixed phase constants is 1/2 a cycle (π or $180°$). If wave 1 travels 37 wavelengths further than wave 2 to a particular point, will the interference at that point be constructive, destructive or partial?

Here the phase difference has two sources: the path length difference *and* the phase factor difference. We could reason through it by saying that the difference in the constant phase ϕ puts them out of phase to begin with, and then shifting by a whole number of wavelengths does not change anything so the intereference is destructive. Let us approach the same problem calculating the total phase.

We know that $x_1 = x_2 + 37\lambda$, as wave 1 travels 37 more wavelengths than wave 2. We also know that ϕ_1 and ϕ_2 are different by π, but we do not know which one is the biggest! The best we can say is that $\phi_1 = \phi_2 \pm \pi$. Then the total phase difference is

$$\begin{aligned}\Delta\Phi &= \frac{2\pi}{\lambda}(x_1 - x_2) + (\phi_1 - \phi_2)\\ &= \frac{2\pi}{\lambda}(x_2 + 37\lambda - x_2) + (\phi_2 \pm \pi - \phi_2)\\ &= \frac{2\pi}{\lambda}37\lambda \pm \pi\\ &= (74 \pm 1)\pi.\end{aligned}$$

If we use the + sign we get $\Delta\Phi = 75\pi$, if use the minus sign we get $\Delta\Phi = 73\pi$ which are both odd multiples of π. Therefore the interference must be destructive.

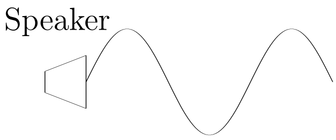
Question 3

This one is easy: the fixed phase constants are the same, so a path difference of 0, λ, 2λ ($,3\lambda, \ldots$) will be constructive. A path difference of $0.5\lambda, 1.5\lambda, 2.5\lambda, \ldots$ will be destructive.

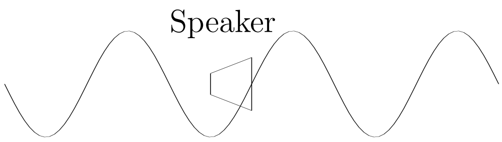
Question 4

A note on the direction of travelling waves

A problem comes about because we need to figure out how to take into account of the fact that the speakers face in opposite directions in two of the examples. Let us consider just one speaker at $x = 0$ facing right. The sound wave to the right is easy:

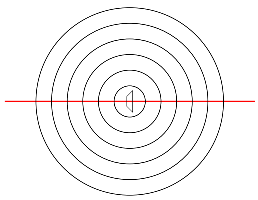


One may guess that the wave to the left would just be a continuation of the sine wave:

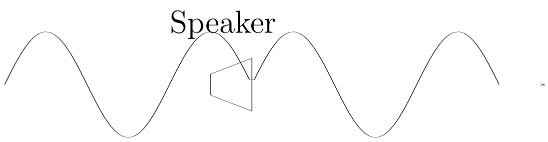


This is wrong!

To see why, we have to remember that the sound waves are really 3-D, even though we are only treating them as 1-D. Consider standing on top of the speaker. The peaks will radiate outwards like the ripples in a pond when a stone is thrown in. In this picture, the black lines represent the minimum amount of pressure (i.e. the lowest point in the sine graph):



Now look at the red line, which represents the simple 1-D model we have. The front and the back are identical, as they are radiating out in circles! It is simple to understand in 2- or 3-D, but in 1-D the profile looks odd:



This is correct!

So it does not matter which way the wave is propogating, and if we just went ahead and calculated we would get the right answer anyway (but for a reason that is quite subtle!).

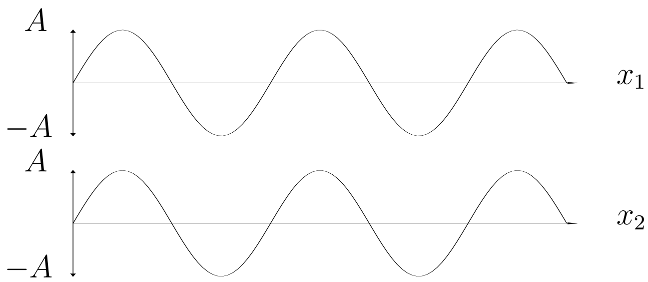
I

Two speakers are facing each other and are wired the *same way.*

The speakers are generating sound from the same place (so we already know that we want graph A or graph D). As we showed in the section above, it does not actually matter which way the two speakers are facing. They are in phase and in the same place (to a very good approximation) so the path lengths are the same. The total change in phase is then (to be very explicit about it)

$$\begin{aligned}\Delta\Phi &= \frac{2\pi}{\lambda}(x_1 - x_2) + (\phi_1 - \phi_2)\\ &= \frac{2\pi}{\lambda}(0) + (0) \qquad (x_1 = x_2 = x,\ \phi_1 = \phi_2 = \phi)\\ &= 0\end{aligned}$$

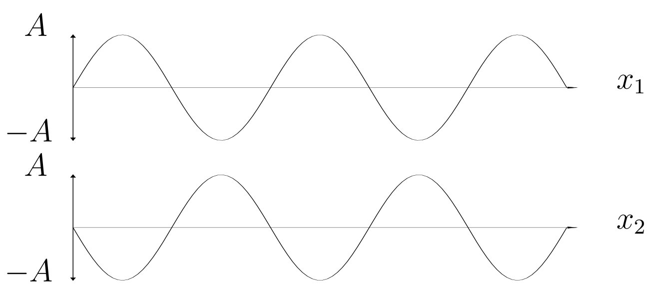
So the two are in phase, which means the answer must be D.



II

Two speakers are facing each other and are wired the *opposite way.*

Everything in this problem is the same except for the wiring. Taking the wiring into account means changing the phase of one or other by π.



so the answer is D.

III

Two speakers are plugged in, wired the same way, both facing you but one speaker is a full wavelength ahead of the other. From a physics point of view, these waves are exactly the same, except that one starts one wavelength after the other. So the answer is clearly E.

To show this via total phase arguments, let us recap the information we have. We know $\phi_1 = \phi_2$ (they are wired the same) and $x_1 = x_2 + \lambda$, so the total phase difference is

$$\begin{aligned}\Phi_1 - \Phi_2 &= \frac{2\pi}{\lambda}(x_1 - x_2) + \phi_1 - \phi_2 \\ &= \frac{2\pi}{\lambda}(x_2 + \lambda - x_2) + \phi_1 - \phi_1 \\ &= 2\pi\end{aligned}$$

which is an even multiple of π, so they look exactly the same as stated above.

IV

Two speakers facing the same way, wired oppositely, both facing you, but one speaker is half a wavelength in front of the other. Is this constructive or destructive interference? Well, we know that we have a path length difference of $\lambda/2$ which is a phase shift of π. We *also* have a phase shift of π from the fact the speakers are wired in opposite directions. So the total phase shift is either 0 or 2π, both of which lead to constructive interference.

Graph B is shifted along half a wavelength, and so is graph C. C is constructive and B is destructive, so the answer must be C.

V

Two speakers facing the same way, wired the same way, both facing you, but one speaker is half a wavelength in front of the other. Same as before, except now we do not have the phase shift of π due to the wiring so the total phase shift is just π. This leads to destructive interference, which is graph B.